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**COMBINING STATE DEPENDENT
RICCATI EQUATION APPROACH
WITH DYNAMIC INVERSION:
APPLICATION TO CONTROL OF
FLIGHT VEHICLES**



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FEBRUARY 2003

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Combining State Dependent Riccati Equation Approach with Dynamic Inversion: Application to Control of Flight Vehicles

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Abstract

State Dependent Algebraic Riccati Equation (SDRE) techniques are rapidly emerging as a design method, which provides a systematic and effective means of designing nonlinear controllers, observers and filters. This paper describes a new method of integrating the SDRE technique with the Dynamic Inversion control law that is frequently used in the design of aircraft control systems. This paper also provides an example by applying this control design technique to a reusable launch vehicle.

Introduction

There have been a number of design methodologies developed for control of nonlinear systems. The aircraft problem is one such nonlinear system to which control design techniques such as Dynamic Inversion have been applied. Lesser-known nonlinear design procedures are those that involve the state dependent Riccati equations (SDRE). The State Dependent Riccati Equation approach to nonlinear system stabilization relies on representing a nonlinear system's dynamics similar to linear dynamics, but with state dependent coefficient matrices that can be inserted into state dependent Riccati equations to generate a feedback law. Although stability of the resulting closed loop system need not be guaranteed a priori, simulation studies have shown that the method can often lead to suitable control laws.

Over the past several years various SDRE design methodologies have been successfully applied to aerospace problems. SDRE based design procedures have been used in advanced guidance law development [1,2] and in an output feedback autopilot design [3]. Additionally, SDRE design methods have been used in nonlinear filter development [4]. In [5], Ehrler and Vadali investigated the nonlinear regulator problem and showed that solving an algebraic Riccati as it evolved over time provided one means of obtaining a sub optimal solution of the infinite horizon problem. In essence the State Dependent Riccati Equation was treated as being time dependent and its state dependency was not explicitly acknowledged, addressed or analyzed. In [6], SDRE nonlinear regulation, SDRE nonlinear H_{∞} , and SDRE nonlinear H_2 design methodologies were defined and the optimality, sub optimality and stability properties of SDRE nonlinear regulation was investigated.

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Overview

SDRE stabilization refers to the use of State Dependent Riccati Equations to construct nonlinear feedback control laws for nonlinear systems. The main idea is to represent the nonlinear system

$$\dot{x} = f(x) + B(x)u$$

in the form

$$\dot{x} = A(x)x + B(x)u$$

and to use the feedback

$$u = -R^{-1}(x)B^T(x)P(x)x$$

where $P(x)$ is obtained from the SDRE

$$P(x)A(x) + A^T(x)P(x) + Q(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) = 0$$

and $Q(\cdot)$ and $R(\cdot)$ are design parameters that satisfy the point wise definiteness condition

$$Q(x) > 0 \quad R(x) > 0$$

The resulting closed loop dynamics have a linear-like structure given by

$$\dot{x} = A_{CL}(x)x \text{ where}$$

$$A_{CL}(x) = A(x) - R^{-1}(x)B(x)B^T(x)P(x)$$

Simulation studies have shown that the dynamics matrix satisfies the Lyapunov Criterion for stability given by

$$P(x)A_{CL}(x) + A_{CL}^T(x)P(x) < -Q_{pd} \text{ where}$$

$$Q_{pd} > 0$$

Equations of Motion of Aircraft

$$\dot{\phi} = P + Q \sin \phi \tan \theta + R \cos \phi \tan \theta$$

$$\dot{\theta} = Q \cos \phi - R \sin \phi$$

$$\dot{\psi} = Q \sin \phi \sec \theta + R \cos \phi \sec \theta$$

$$\dot{P} = c_1 R Q + c_2 P Q + c_3 L + c_4 N$$

$$\dot{Q} = c_5 P R - c_6 (P^2 - R^2) + c_7 M$$

$$\dot{R} = c_8 P Q - c_2 R Q + c_4 L + c_9 N$$

$$\dot{U} = R V - Q W + \frac{F_x}{m} - g \sin \theta$$

$$\dot{V} = -R U + P W + \frac{F_y}{m} + g \cos \theta \sin \phi$$

$$\dot{W} = Q U - P V + \frac{F_z}{m} + g \cos \theta \cos \phi$$

where

$$c_1 = [(J_y - J_z)J_z - J_{xz}^2]/\Gamma$$

$$c_2 = (J_x - J_y + J_z)J_{xz}/\Gamma$$

$$c_3 = J_z/\Gamma$$

$$c_4 = J_{xz}/\Gamma$$

$$c_5 = (J_z - J_x)/J_y$$

$$c_6 = J_{xz}/J_y$$

$$c_7 = 1/J_y$$

$$c_8 = [(J_x - J_y)J_x + J_{xz}^2]/\Gamma$$

$$c_9 = J_x/\Gamma$$

where

$$\Gamma = J_x J_z - J_{xz}^2$$

State Dependent Linear State Space System

Example 1

State : $x = [P \ Q \ R]$

$$\dot{x} = \begin{bmatrix} c_2x_2 & c_1x_3 & 0 \\ -c_6x_1 & 0 & c_5x_1 + c_6x_3 \\ 0 & c_8x_1 & -c_2x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} c_3 & 0 & c_4 \\ 0 & c_7 & 0 \\ c_4 & 0 & c_9 \end{bmatrix} \begin{bmatrix} L \\ M \\ N \end{bmatrix}$$

Example 2

State : $x = [P \ Q \ R \ \phi \ \theta \ \psi]$

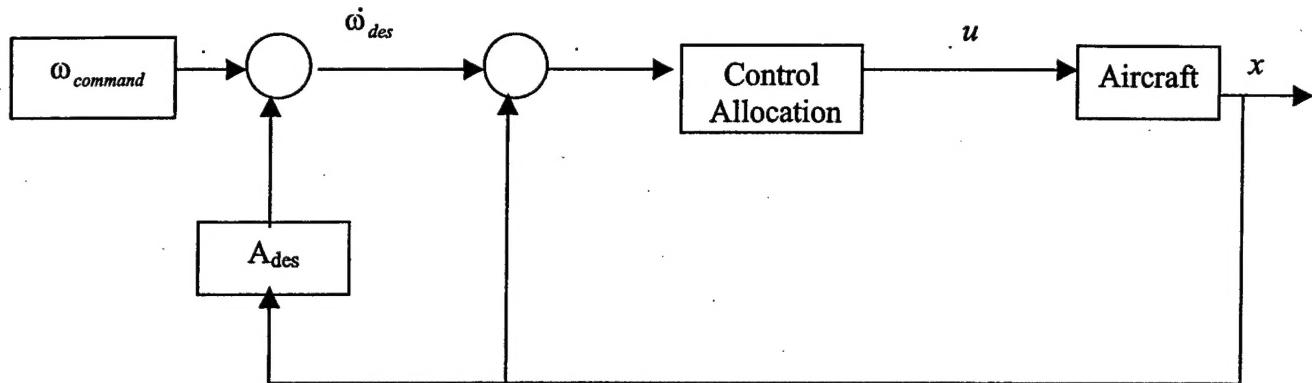
$$\dot{x} = \begin{bmatrix} c_2x_2 & c_1x_3 & 0 & 0 & 0 & 0 \\ -c_6x_1 & 0 & c_5x_1 + c_6x_3 & 0 & 0 & 0 \\ 0 & c_8x_1 & -c_2x_2 & 0 & 0 & 0 \\ 1 & \sin x_5 \tan x_5 & \cos x_5 \tan x_5 & 0 & 0 & 0 \\ 0 & \cos x_4 & -\sin x_4 & 0 & 0 & 0 \\ 0 & \sin x_4 \sec x_5 & \cos x_4 \sec x_5 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} c_3 & 0 & c_4 \\ 0 & c_7 & 0 \\ c_4 & 0 & c_9 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} L \\ M \\ N \end{bmatrix}$$

Example 3

State : $x = [U \ V \ W \ P \ Q \ R \ \sin\theta \ \cos\theta \sin\phi \ \cos\theta \cos\phi]$

$$\dot{x} = \begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 & -g & 0 & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 \\ 0 & 0 & 0 & c_2x_5 & c_1x_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_6x_4 & 0 & c_5x_4 + c_6x_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_8x_1 & -c_2x_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x_6 & x_5 & x_7 \\ 0 & 0 & 0 & 0 & 0 & x_6 & 0 & x_4 & x_8 & x_9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} + \begin{bmatrix} 1/m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & 0 & c_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_4 & 0 & c_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F \\ F \\ F \\ L \\ M \\ N \end{bmatrix}$$

Schematic Representation of the Method



Combined SDRE and Dynamic Inversion Control Law

Stability of Nominal System for Full State Feedback

$$\dot{x} = A(x)x + Bu$$

Riccati Based Control Law

$$u = -R^{-1}B^T P(x)x$$

Dynamic Inversion Control Law

$$u = B^{-1}(\dot{x}_{des} - A(x)x)$$

$$W.K.T \quad B^{-1} = (B^T B)^{-1}$$

Let

$$\dot{x}_{des} = A_{des}(x)x$$

Then

$$u = -(B^T B)^{-1} B^T (A(x) - A_{des}(x))x$$

Comparing Equations (1) & (2)

$$R = B^T B$$

$$P(x) = A(x) - A_{des}(x)$$

The SDARE becomes

$$P(x)A(x) + A^T(x)P(x) + Q - P(x)P(x) = 0$$

Solving the above equation for $P(x)$, we can calculate

$$A_{des}(x) = A(x) - P(x)$$

Stability of Nominal System for Output Feedback

$$\dot{x} = A(x)x + Bu$$

$$\omega = Cx$$

$$\dot{\omega} = C\dot{x} = CA(x)x + CBu$$

$$\text{Let } \omega_{des} = A_{des}(x)x$$

Riccati Based Control Law

$$u = -R^{-1}B^T P(x)x$$

Dynamic Inversion Control Law

$$u = (CB)^+ (\omega_{des} - CA(x)x)$$

$$u = -(CB)^+ (CA(x) - A_{des}(x))x$$

$$u = -\{(CB)^T (CB)\}^{-1} (CB)^T (CA(x) - A_{des}(x))x$$

$$u = -(B^T C^T CB)^{-1} B^T C^T (CA(x) - A_{des}(x))x$$

Comparing Equations (3) & (4)

$$R = B^T C^T CB$$

$$P(x) = C^T (CA(x) - A_{des}(x))$$

The SDARE becomes

$$P(x)A(x) + A^T(x)P(x) + Q - P(x)B(B^T C^T CB)^{-1} B^T P(x) = 0$$

Solving the above equation for $P(x)$, we can calculate A_{des} using the equation

$$P(x) = C^T (CA(x) - A_{des}(x))$$

Closed Loop System

$$\dot{x} = [A(x) + B(CB)^+ (A_{des}(x) - A(x))]x = A_c(x)x$$

Verification of Stability

Let P_L be a positive definite matrix, which is chosen as $P_L = P(x_0)$. $P(x_0)$ is the solution to the SDARE at the initial condition $\{x_0\}$.

Full State Feedback

$$P(x_0)A(x_0) + A^T(x_0)P(x_0) + Q - P(x_0)P(x_0) = 0$$

Output Feedback

$$P(x_0)A(x_0) + A^T(x_0)P(x_0) + Q - P(x_0)B(B^T C^T C B)^{-1} B^T P(x_0) = 0$$

The Closed Loop System is locally asymptotically stable if

$$P_L A_c(x) + A_c^T(x) P_L < 0$$

Application of Combined SDRE and Dynamic Inversion Control Law to Aircraft

$$\text{State : } x = [U \ V \ W \ P \ Q \ R \ \sin\theta \ \cos\theta\sin\phi \ \cos\theta\cos\phi]$$

$$\dot{x} = \begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 & -g & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 & g & 0 & 0 \\ 0 & 0 & 0 & c_2 x_5 & c_1 x_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_6 x_4 & 0 & c_5 x_4 + c_6 x_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_8 x_1 & -c_2 x_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -x_6 & x_5 & x_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_6 & 0 & x_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & -x_5 & -x_4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} + \begin{bmatrix} 1/m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & 0 & c_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_4 & 0 & c_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \\ L \\ M \\ N \end{bmatrix}$$

where

$$A(x) = \begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 & -g & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 & g & 0 & 0 \\ 0 & 0 & 0 & c_2 x_5 & c_1 x_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_6 x_4 & 0 & c_5 x_4 + c_6 x_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_8 x_1 & -c_2 x_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -x_6 & x_5 & x_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_6 & 0 & x_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & -x_5 & -x_4 & 0 \end{bmatrix} \quad \& B = \begin{bmatrix} 1/m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & 0 & c_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_4 & 0 & c_9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The above state space system is not completely controllable. However if we separate the $A(x)$ matrix into $A_1(x)$ and $A_2(x)$ given by

$$A_1(x) = \begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2x_5 & c_1x_6 & 0 \\ 0 & 0 & 0 & -c_6x_4 & 0 & c_5x_4 + c_6x_6 \\ 0 & 0 & 0 & 0 & c_8x_1 & -c_2x_5 \end{bmatrix} \quad \text{and} \quad A_2(x) = \begin{bmatrix} 0 & -x_6 & x_5 \\ x_6 & 0 & x_4 \\ -x_5 & -x_4 & 0 \end{bmatrix}$$

then the pair $(A_1(x), B_1)$ are completely controllable.

$$A_1(x) = \begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2x_5 & c_1x_6 & 0 \\ 0 & 0 & 0 & -c_6x_4 & 0 & c_5x_4 + c_6x_6 \\ 0 & 0 & 0 & 0 & c_8x_1 & -c_2x_5 \end{bmatrix} \quad \text{and} \quad B_1 = \begin{bmatrix} 1/m & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3 & 0 & c_4 \\ 0 & 0 & 0 & 0 & c_7 & 0 \\ 0 & 0 & 0 & c_4 & 0 & c_9 \end{bmatrix}$$

Therefore we can design a control law given by

$$u = B_1^{-1}(\dot{x}_{des} - A_1(x)x_1)$$

$$\text{where } x_1 = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6] \\ \text{and } u = [F_x \ F_y \ F_z \ L \ M \ N]$$

$$W.K.T \quad B_1^{-1} = (B_1^T B_1)^{-1}$$

Let

$$\dot{x}_{1des} = A_{1des}(x)x_1$$

Then

$$u = -(B_1^T B_1)^{-1} B_1^T (A_1(x) - A_{1des}(x))x_1$$

$A_{1des}(x)$ is calculated from the equation

$$A_{1des}(x) = A_1(x) - P(x)$$

where $P(x)$ is the solution to the Riccati Equation

$$P(x)A_1(x) + A_1^T(x)P(x) + Q - P(x)B_1(x)R^{-1}(x)B_1^T(x)P(x) = 0$$

However we know that

$$R = B_1^T B_1$$

Therefore the State Dependent Algebraic Riccati Equation reduces to

$$P(x)A_1(x) + A_1^T(x)P(x) + Q - P(x)P(x) = 0$$

The closed loop system is given by

$$\dot{x}_1 = A_1(x)x_1 + B_1[B_1^{-1}(A_{1des}(x) - A_1(x))x_1]$$

Therefore, we have

$$\dot{x}_1 = A_{cl}(x)x_1$$

$$\text{where } A_{cl}(x) = A_1(x) + B_1[B_1^{-1}(A_{1des}(x) - A_1(x))]$$

Verification of Closed Loop System Stability

In the previous section we separated $A(x)$ into

$$A_1(x) = \begin{bmatrix} 0 & x_6 & -x_5 & 0 & 0 & 0 \\ -x_6 & 0 & x_4 & 0 & 0 & 0 \\ x_5 & -x_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2x_5 & c_1x_6 & 0 \\ 0 & 0 & 0 & -c_6x_4 & 0 & c_5x_4 + c_6x_6 \\ 0 & 0 & 0 & 0 & c_8x_1 & -c_2x_5 \end{bmatrix} \quad \text{and} \quad A_2(x) = \begin{bmatrix} 0 & -x_6 & x_5 \\ x_6 & 0 & x_4 \\ -x_5 & -x_4 & 0 \end{bmatrix}$$

The matrix $A_2(x)$ is neutrally stable if we calculate it's eigenvalues by freezing the states at each time instant. However we are more concerned about the stability of matrix $A_1(x)$ under the control 'u' that we previously discussed. The closed loop system under control 'u' is given by $A_{cl}(x)$.

Let P_L be a positive definite matrix, which is chosen as $P_L = P(x_{10})$. $P(x_{10})$ is the solution to the SDARE at the initial condition $\{x_{10}\}$.

$$P(x_{10})A(x_{10}) + A^T(x_{10})P(x_{10}) + Q - P(x_{10})P(x_{10}) = 0$$

The closed loop system is locally asymptotically stable if

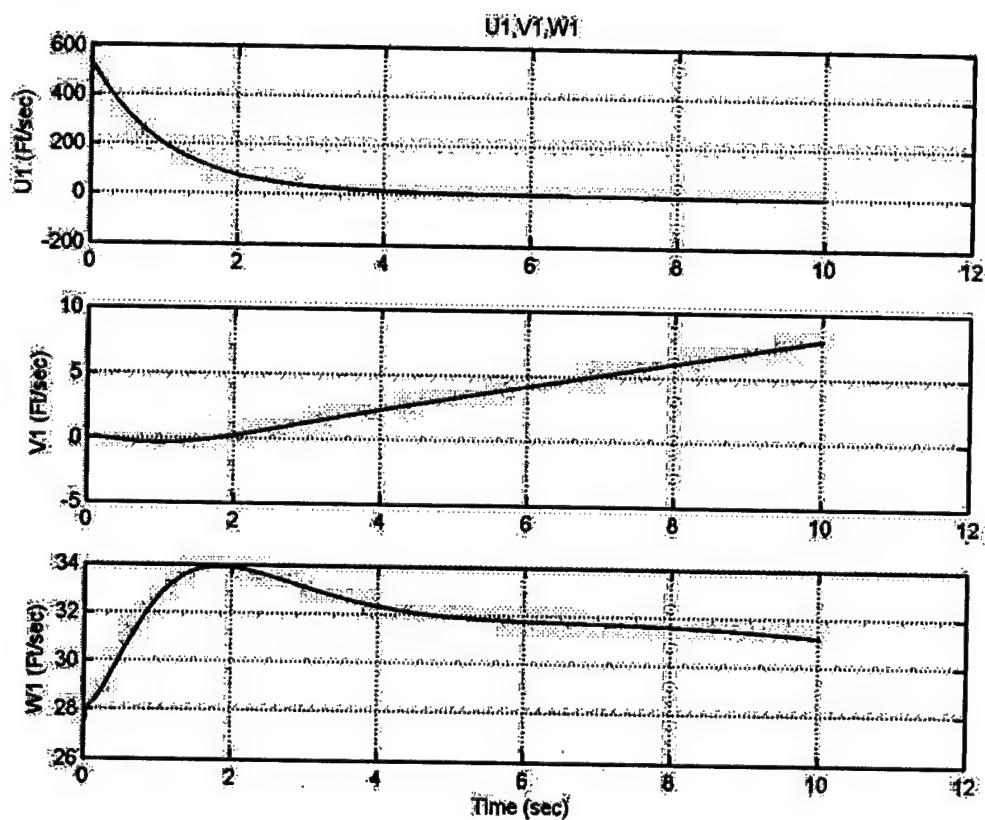
$$P_L A_{cl}(x) + A_{cl}^T(x)P_L < 0$$

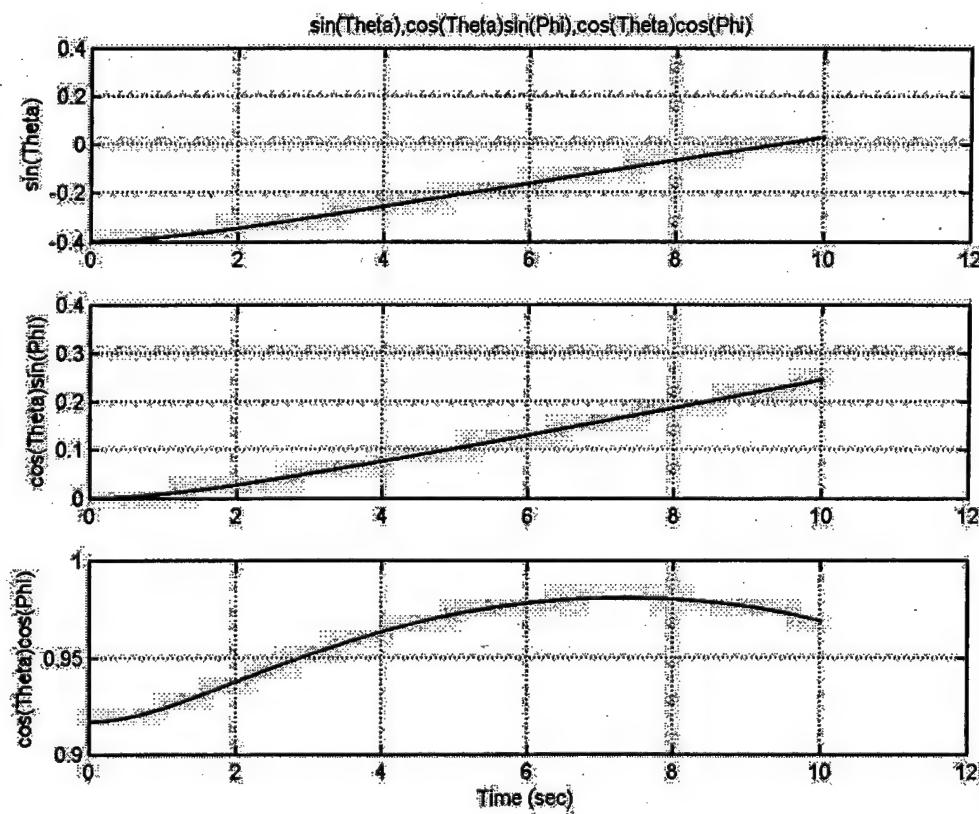
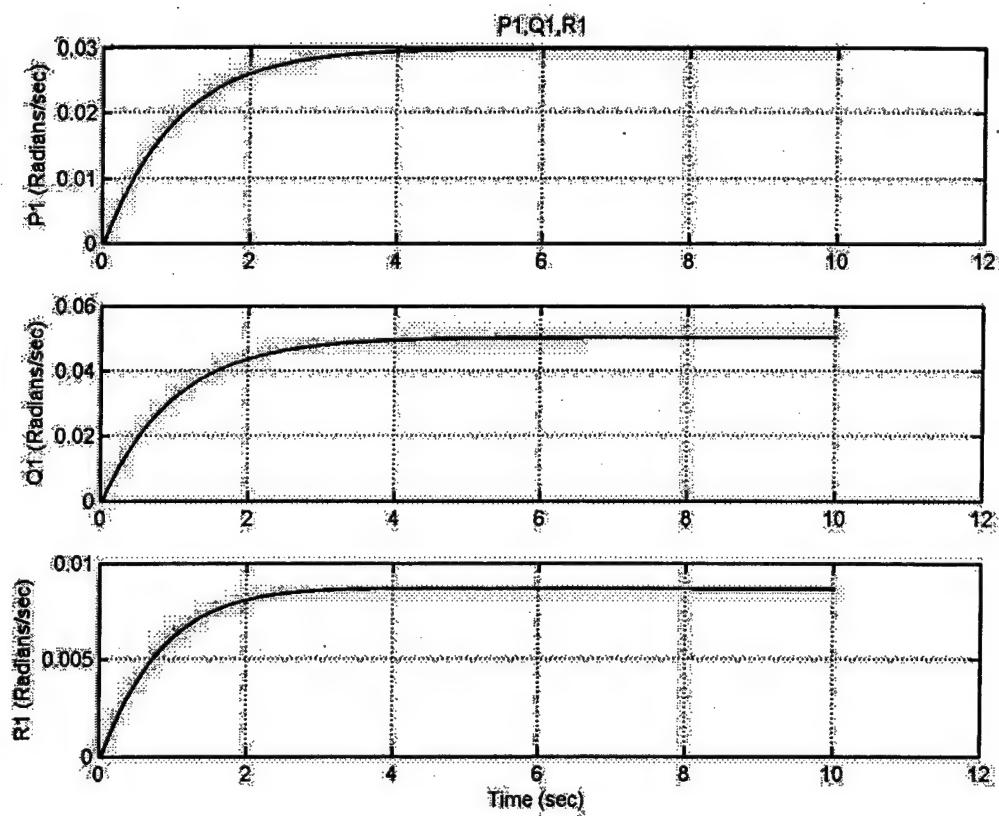
Results

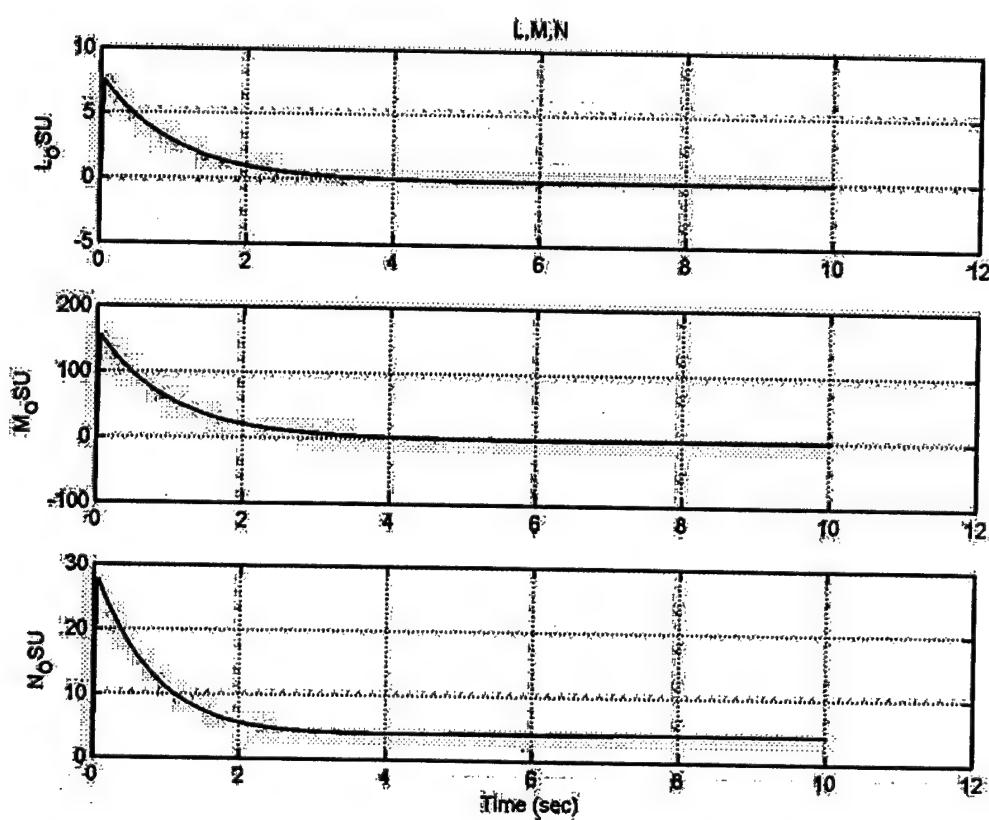
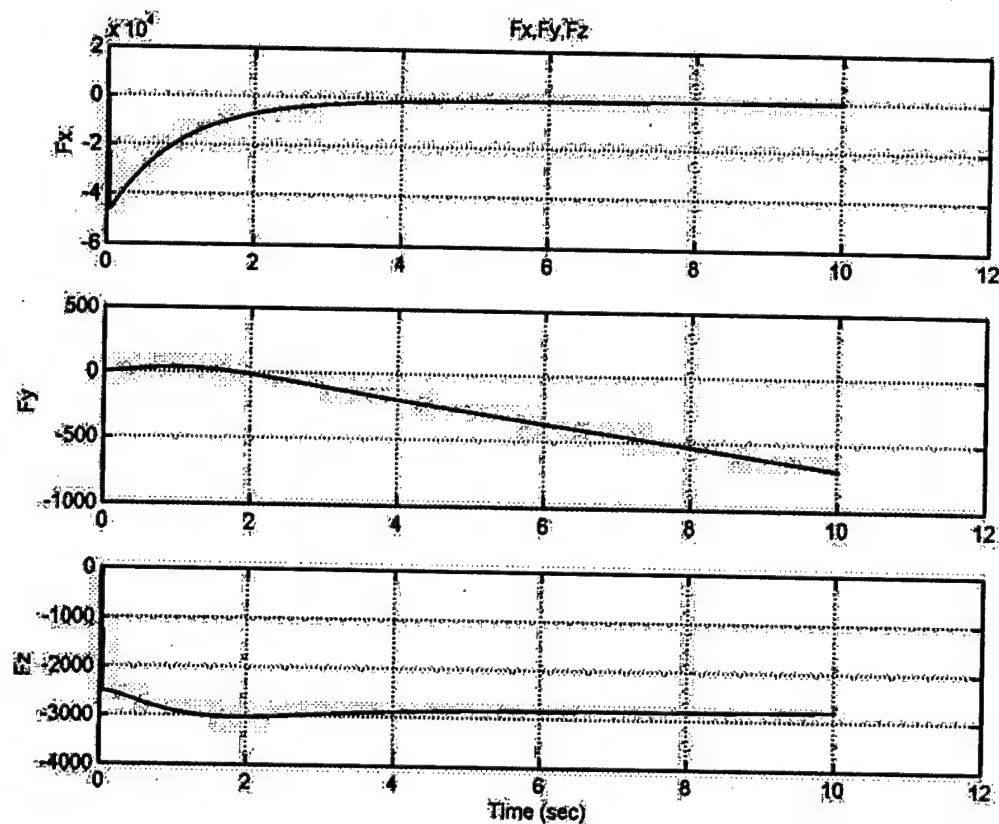
Initial Condition 1

$$U_0 = 517.5$$

$V_0 = 0$
 $W_0 = 27.5$
 $P_0 = 0$
 $Q_0 = 0$
 $R_0 = 0$







Results

Initial Condition 2

$U_0 = 517.5$

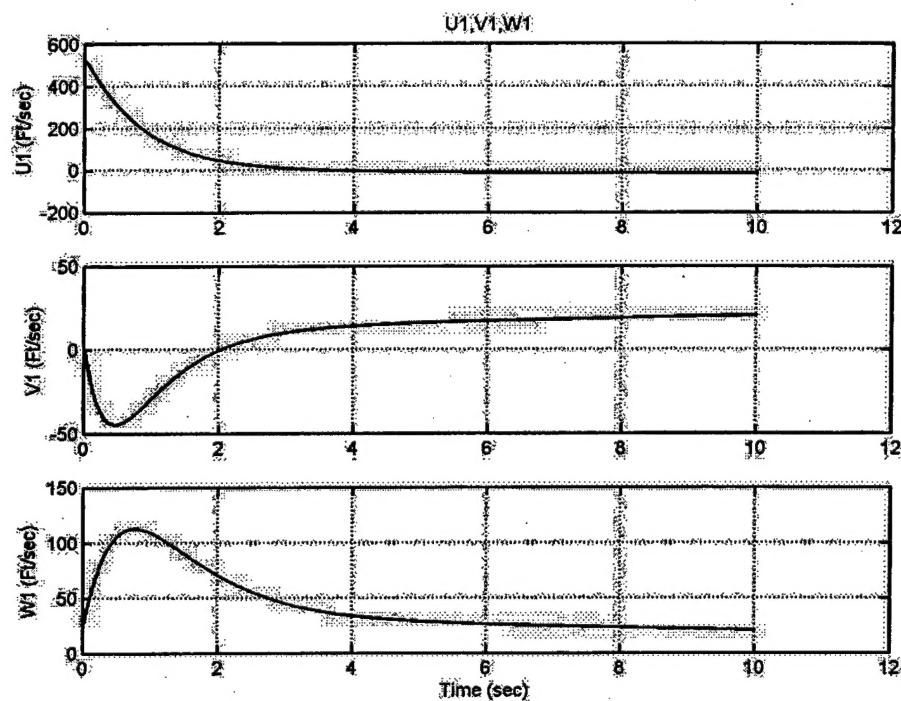
$V_0 = 0$

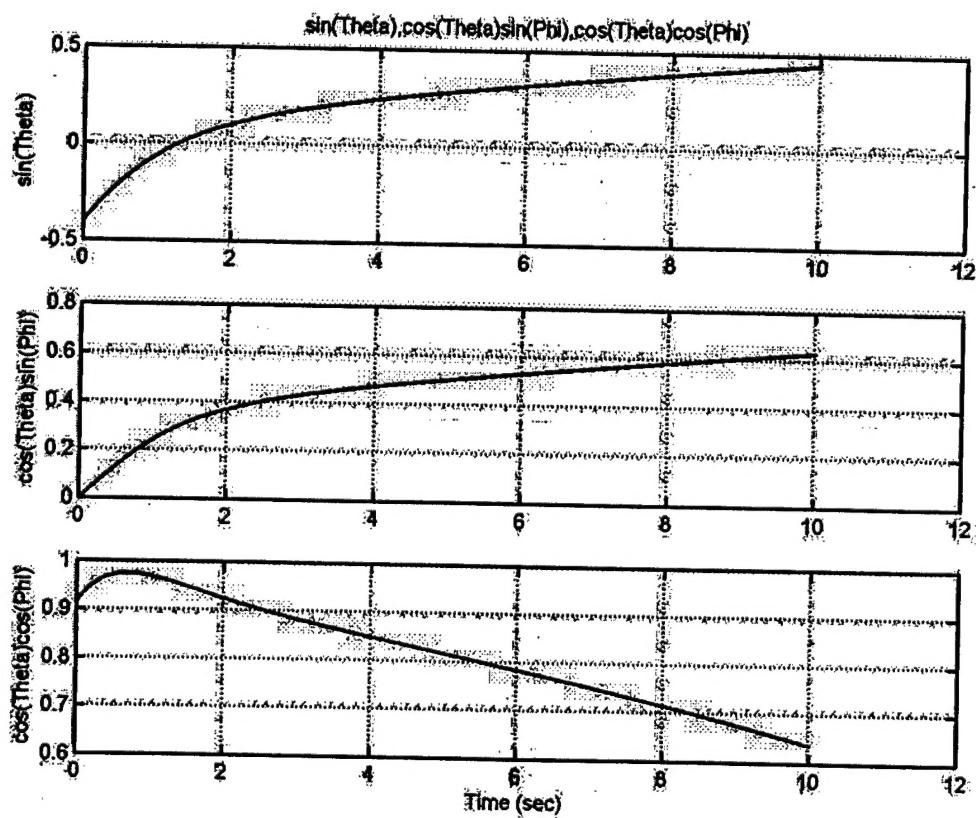
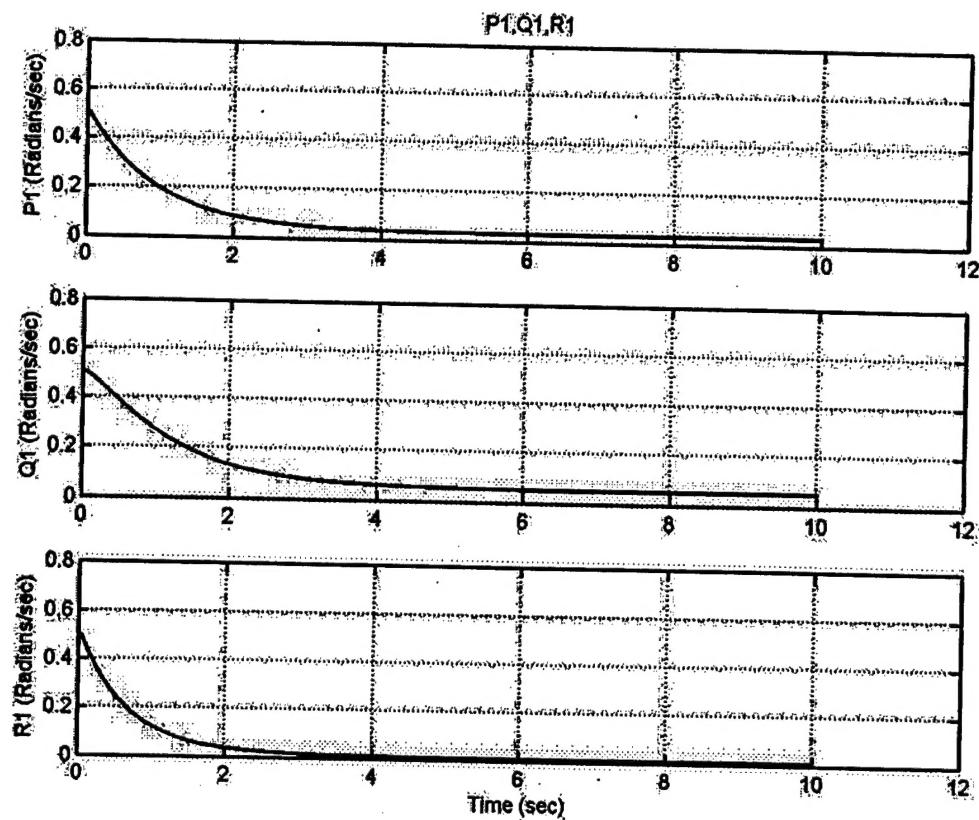
$W_0 = 27.5$

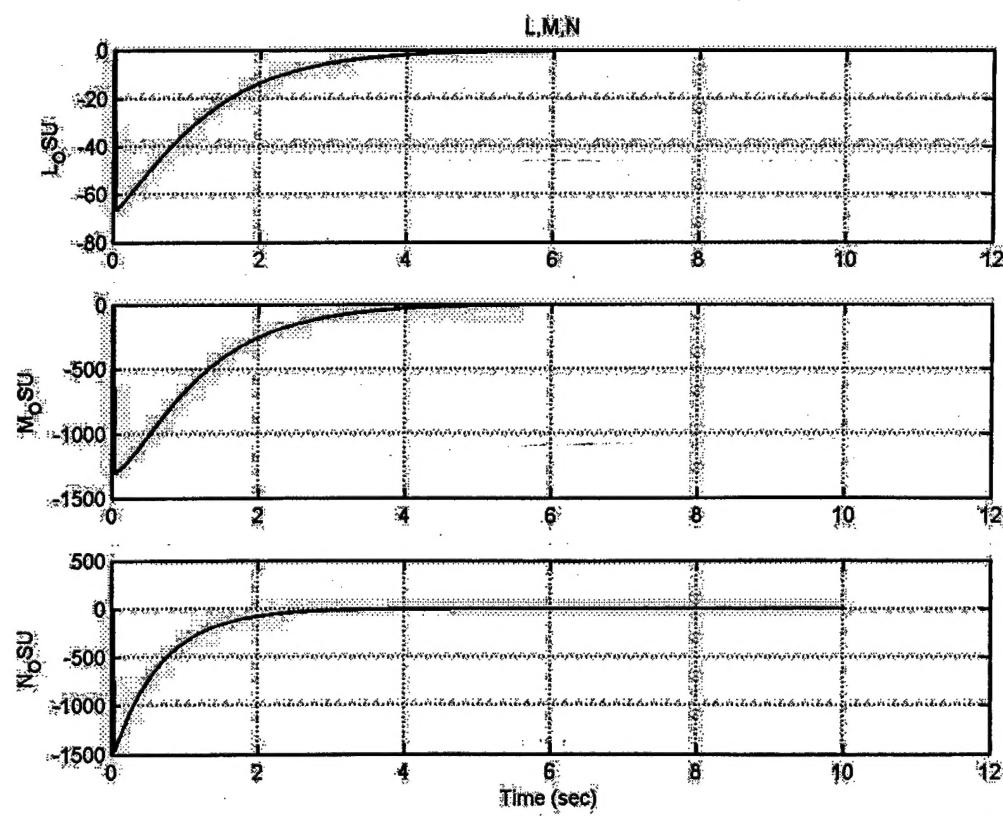
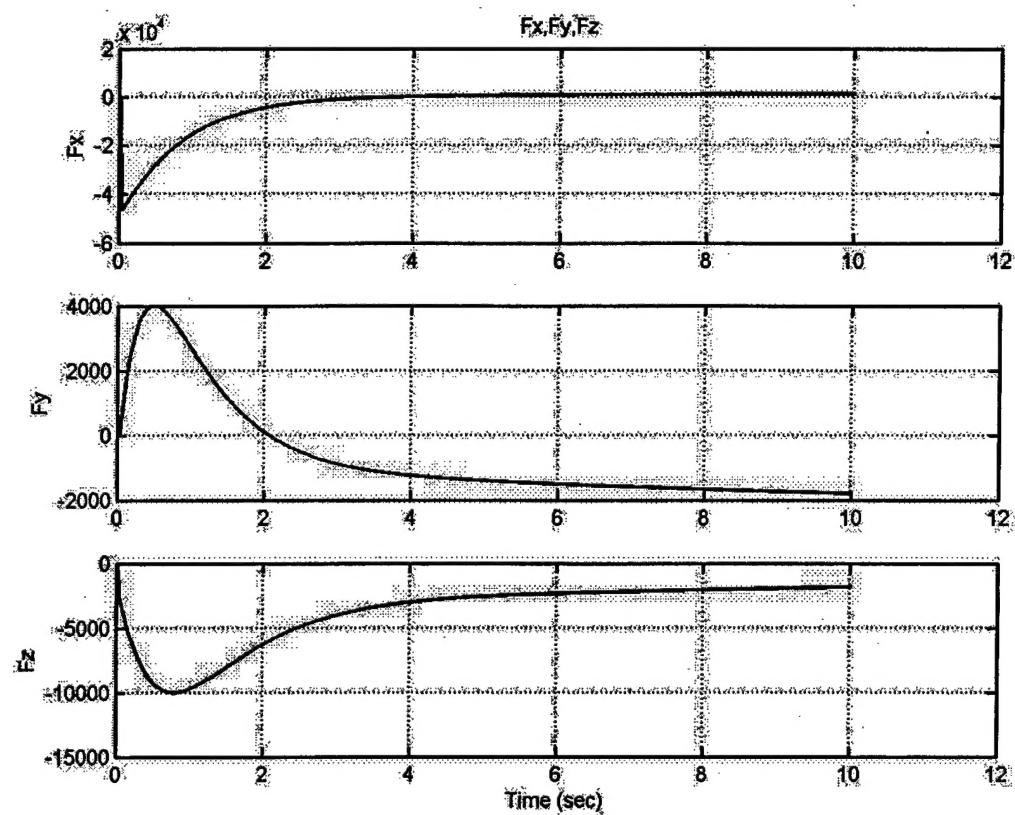
$P_0 = 0.5$

$Q_0 = 0.5$

$R_0 = 0.5$







Conclusions

In this paper a new control law was developed that is a combination of the existing State Dependent Riccati Equation techniques and the Dynamic Inversion control law. This control system design was then applied to the aircraft dynamics and the resulting closed loop system was shown to be stable even under change in the initial conditions.

References

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